PHYSICAL REVIEW D, VOLUME 70, 101301(R)

Scaling dark energy

Salvatore Capozziello,[1] Alessandro Melchiorri,[2] and Alice Schirone[1]
[1]*Dipartimento di Fisica "E.R. Caianiello," Università di Salerno and INFN, Sez. di Napoli, Gruppo Coll. di Salerno, Via S. Allende, 84081, Baronissi (Salerno), Italy*
[2]*Dipartimento di Fisica "G. Marconi," Università di Roma "La Sapienza" and INFN, Sez. di Roma, Ple Aldo Moro 5, 00185, Roma, Italy*
(Received 11 August 2004; published 11 November 2004)

We investigate the possibility that dark energy is scaling with epochs. A phenomenological model is introduced whose energy density depends on the redshift z in such a way that a smooth transition among the three dominant phases of the Universe evolution (radiation era, matter domination, asymptotic de Sitter state) is achieved. We use the Wilkinson Microwave Anisotropy Probe cosmic microwave background data and the luminosity distances of Type Ia Supernovae to test whether the model is in agreement with astrophysical observations.

DOI: 10.1103/PhysRevD.70.101301 PACS numbers: 98.80.–k, 98.65.–r, 98.70.Vc

I. INTRODUCTION

One of the greatest challenges in modern cosmology is to identify the nature of the dark energy component which is causing the observed accelerated expansion of the Universe. Since a cosmological constant, while in agreement with current observations, is theoretically flawed, several alternatives have been proposed. Some of the popular candidates to explain the observations are a slowly-rolling scalar field, "quintessence" [1,2], or a "k-essence" scalar field with noncanonical kinetic terms in the Lagrangian [3,4], or "coupled quintessence" where the scalar field is nonminimally coupled with gravity [5]. The cosmological acceleration can be also achieved considering geometrical counterparts in the gravitational Lagrangian other than the standard Ricci scalar of general relativity. This fact allows us to define a sort of effective curvature pressure and curvature energy density which act as a time-varying cosmological constant [6].

One way to distinguish whether the dark energy is due to a cosmological constant, to a scalar field, or something else is to measure the equation of state w_X, the ratio of the pressure p_X to the energy density ρ_X. A cosmological constant always has $w_X = -1$ while scalar fields or curvature counterparts generally have an equation of state which differs from unity and varies with redshift z. Through measurements of Type Ia Supernovae (SN-Ia), large-scale structure and cosmic microwave background (CMB) anisotropies, the equation of state may be determined accurately enough in the next few years to find out whether dark energy is actually different from a cosmological constant or not.

An important general property of these so-called "tracker" models [7] (and of any dark energy model which aims to alleviate the fine tuning problem of the cosmological constant) is that the scalar field equation of state (and its energy density) remains close to that of the dominant background component during most of the cosmological evolution.

For example, in power-law potential like $V = V_0/\phi^\alpha$ the equation of state generally remains closer to the background value $w_X = \alpha w_B/(\alpha + 2)$ while the ratio of the energy density of the scalar field to that of the dominant component gradually increases. In models based on exponential potential $V = V_0 e^{-\lambda\phi}$ [8,9] $w_X(z)$ mimics exactly the scaling of the dominant background in the attractor regime ($w_X = w_B$) and if the background component scales as $\rho_B = \rho_0(\frac{a_0}{a})^n$, then the scaling field approaches an attractor solution, and its fractional energy density is given by $\Omega_X = \frac{n}{\lambda^2}$, constant with redshift. As further example, in "k-essence" models, the k-essence undergoes two transitions in its behavior, one at the onset of matter domination and a second when k-essence begins to dominate over the matter density. During the radiation-dominated era, the k-essence energy tracks the radiation, falling as $1/a^4$ where a is the scale factor. The onset of the matter-dominated era automatically triggers a change in the behavior of k-essence such that it begins to act as an energy component with $w_X(z) \leq 0$. When k-essence overtakes the matter density, $w_X(z)$ changes to another value around -1, the precise value of which depends on the detailed model.

Given the new release of cosmological data from high-precision measurements of CMB anisotropies (see, e.g., [10]) and SN-Ia luminosity distances [11] (which are now providing a $\sim 18\sigma$ evidence for a dark energy component) is therefore extremely timely to check if any hint for a "*scaling*" dark energy is present in the data.

Moreover, recent analysis of SN-Ia data (see, e.g., [12,13]) with model independent parametrization have found that dark energy which evolves with time provides a better fit to the SN-Ia data than a standard cosmological constant.

In this paper we use a phenomenological approach to constrain a dark energy component with an evolutionary behavior similar to the models mentioned above. In particular, we use a toy model whose energy density depends on the redshift z in such a way that a smooth transition among the main three cosmological scaling regimes (radiation, matter, and dark energy) is achieved [14]. We then use the Wilkinson Microwave Anisotropy Probe (WMAP) CMB data and the luminosity distances of SN-Ia to test whether the model is in agreement with astrophysical observation and/or any evidence for "scaling" dark energy is present in the data.

This approach has the main advantage of being theoretically well motivated since a scaling model can reasonably approximate the behavior of most of the dark energy theories on the market (cosmological constant included). An analysis with a higher number of parameters to describe the dark energy evolution would probably be better suited to detect variations from a cosmological constant or to test models with a rapidly evolving equation of state. However, allowing more degrees of freedom could introduce serious degeneracies, fit unknown systematics, and produce final results of difficult theoretical interpretation. The analysis and results presented here can be therefore considered as complementary to recent analysis which sampled a wider set of parameters (see, e.g., [15]) or on the contrary restricted the study to a constant with redshift equation of state (see, e.g., [10,16]). The plan of the paper is as follows: in the next section, the phenomenological model is illustrated in detail. In Sec. III, the analysis method is briefly discussed while in Sec. IV we present the results. Section V is devoted to conclusions.

II. A PHENOMENOLOGICAL MODEL FOR SCALING DARK ENERGY

Let us now illustrate in detail the phenomenological method used for our analysis. The energy density of our scaling dark energy model evolves with redshift as (see [14]):

$$\rho_X(z) = A\left(1 + \frac{1+z}{1+z_s}\right)\left[1 + \left(\frac{1+z}{1+z_b}\right)^3\right] \tag{1}$$

where A is a normalization constant, related to the today dark energy density Ω_X, and z_s and z_b are two free parameters that identify the three epochs of scaling. The corresponding dark energy equation of state is indeed:

$$w_X(z) = \frac{[(\frac{1+z}{1+z_b})^3 - 2]\frac{1+z}{1+z_s} - 3}{3(1 + \frac{1+z}{1+z_s})[1 + (\frac{1+z}{1+z_b})^3]}, \tag{2}$$

which depends on the parameters z_s and z_b such that:

$$w_X \sim 1/3 \quad \text{for } z \gg z_s,$$

$$w_X \sim 0 \quad \text{for } z_b \ll z \ll z_s,$$

$$w_X \sim -1 \quad \text{for } z \ll z_b.$$

Therefore the model we obtain is able to mimic a fluid following first a radiation equation of state, then a matter phase, and finally approaching a de Sitter phase with constant energy. This model is useful to identify and/or constrain a cosmological imprint of scaling dark energy in the data. The same model can be further extended to a fluid with transitions between more generic equations of state. Similar two-parameter dark energy models (the present equation of state and the sound speed) have been already proposed in [17].

In Fig. 1, the behavior of the energy density of the three components in our model (matter, radiation, and dark energy) as function of the scale factor are plotted. As one can see, if the matter to radiation transition redshift is much smaller than the Λ-CDM redshift of equivalence $z_s < z_{Eq} \sim 3200$, the dark energy can be again dominant in the past. However this can be tuned by z_b which shifts the dark energy transition between a cosmological constant and matter. For example, a radiation to matter transition at redshift $z_s \sim 100$ can still be in agreement with a negligible dark energy contribution in the past, providing that $z_b > 5$.

The time variation in w_X is small in comparison to the expansion rate of the Universe. We assume purely adiabatic contributions to the perturbations in the spectrum. The sound speed is therefore fully determined by $w_X(z)$ and we integrate the evolution equations for the density and velocity (we neglect shear) perturbations in the dark energy fluid as in [18]. The adiabatic approximation can be considered as a good start for analyzing the current data (see, e.g., [19,20]).

In Fig. 2, top panel, we plot several power spectra computed with CMBFAST [21] as function of z_b with matter density $\Omega_m = 0.35$, $\Omega_X = 0.65$ and the dark energy matter-radiation transition redshift fixed at $z_s = 5000$, well before the redshift of equivalence in standard cold dark matter model dominated by a cosmological constant (Λ-CDM). As we can see, increasing z_b has the effect of mimicking more and more the cosmological constant behavior. On the other hand, decreasing z_b increases the effective equation of state $w_{\rm eff}$ shifting the peaks in the CMB spectrum toward smaller angular scales (see, e.g., [22]).

In Fig. 2, bottom panel, $z_b = 5$ is fixed and the dependence of z_s is studied. In this case, the effect is smaller, and it is clear that as soon as z_s becomes smaller than the redshift of equivalence in Λ-CDM, Ω_X can dominate again as a relativistic component and leave an imprint on the CMB spectrum through the Early Integrated Sachs-Wolfe effect (see, e.g., [23]).

III. THE ANALYSIS METHOD

In order to bound this phenomenological dark energy model, we consider a template of flat, adiabatic, X-CDM models computed with CMBFAST [21]. We sample the

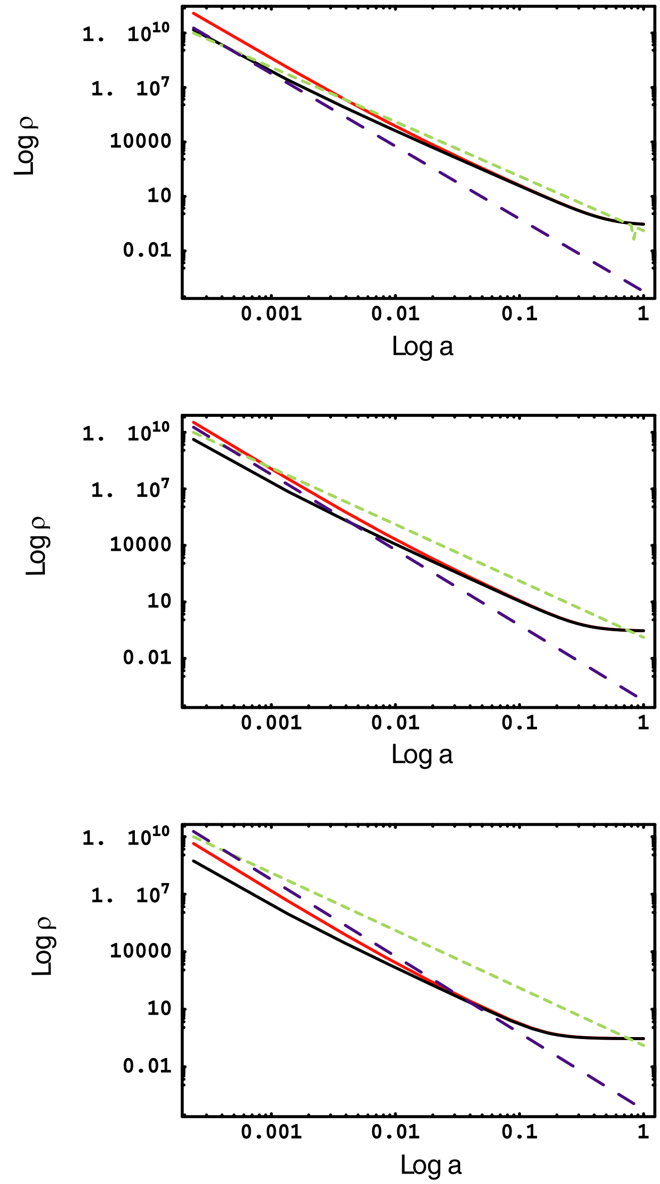

FIG. 1 (color online). Evolution of the overall energy density of the Universe with redshift in different theoretical frameworks. In the panels, the phenomenological dark energy model described in the text has $z_s = 100$ (gray line) and $z_s = 1000$ (black line). The top, middle, and bottom panels show the cases with $z_b = 1$, $z_b = 2$ and $z_b = 5$, respectively. In all plots, the short-dashed line is the matter component while the long-dashed line is radiation (photons and three massless neutrinos) contribution.

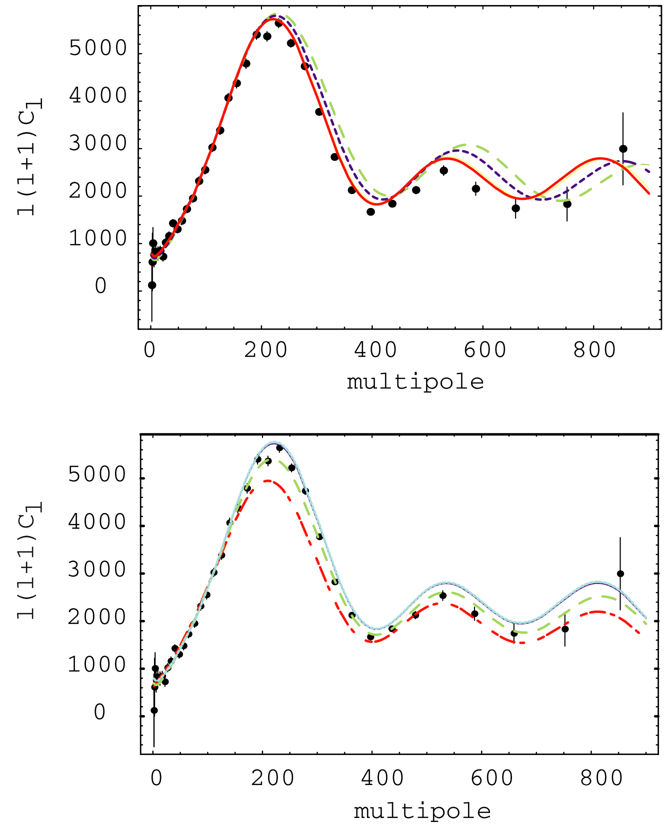

FIG. 2 (color online). Top panel: CMB power spectra normalized at $\ell = 111$ for a Universe with $\Omega_m = 0.35$ and $\Omega_X = 0.65$. The transition redshift matter-radiation has been fixed to $z_s = 5000$. Models with different values of z_b are plotted: solid red line ($z_b = 1$), short-dashed blue line ($z_b = 2$), dashed green line ($z_b = 5$) . Bottom panel: CMB power spectra normalized at $\ell = 111$ for a Universe with $\Omega_m = 0.35$ and $\Omega_X = 0.65$. The matter-dark energy transition redshift has been fixed to $z_b = 5$. Models with different values of z_s are plotted: solid blue line ($z_s = 5000$), dashed green line ($z_s = 100$), short-dashed red line ($z_s = 5$). The WMAP data points are also plotted for comparison.

relevant parameters as follows: $\Omega_{\rm cdm}h^2 = 0.05, \ldots, 0.20$, in steps of 0.01; $\Omega_b h^2 = 0.015, \ldots, 0.030$ (motivated by big bang nucleosynthesis), in steps of 0.001 and $\Omega_X = 0.05, \ldots, 0.95$, in steps of 0.02.

We sample the two parameters of the dark energy model in the range $z_b = 0.05, \ldots, 7.55$, in steps of 0.5 and $z_s = 20, \ldots, 740$ in steps of 80.

The value of the Hubble constant in our database is not an independent parameter, since it is determined through the flatness condition. The conservative top-hat bound $0.55 < h < 0.85$ is adopted and the 1σ constraint on the Hubble parameter, $h = 0.71 \pm 0.07$, obtained from Hubble Space Telescope (HST) measurements [24], is also considered.

We allow for a reionization of the intergalactic medium by varying the Compton optical depth parameter τ_c over the range $\tau_c = 0.05, \ldots, 0.25$ in steps of 0.02. We also vary the spectral index of primordial fluctuations in the range $n_s = 0.80, \ldots, 1.30$ in steps of 0.005. Including variations in τ_c and n_S is rather important since as it has been shown in [15,25] the Integrated Sachs-Wolfe effect produced by the late time dark energy dynamics enlarges the degeneracy between $n_s - \tau_c$ and slightly $n_s - \omega_b h^2$.

For the CMB data, we take into account the recent temperature and cross polarization results from the

WMAP satellite [10] using the method explained in [26] and the publicly available code on the LAMBDA web site.

Finally constraints obtained from the luminosity measurements of Type Ia Supernovae are incorporated from [11] using the GOLD dataset. The SN-Ia luminosity data are helpful in breaking degeneracies between the parameters we are going to consider.

IV. THE RESULTS

In Fig. 3, the likelihood contours are plotted in the (z_s, z_b) plane only for the CMB data. As we can see the current data does not show any evidence for scaling dark energy and we can only derive weak lower limits on the two parameters of the model. We have found that $z_s > 60$ and $z_b > 3.8$ at $1-\sigma$. From the same figure it is clear that an interesting correlation exists between the two parameters. Namely, the current CMB data do not favor or provide evidence for an extra matter or radiation component: if z_s is too small then it is necessary to consider larger values of z_b in order to have the extra dark energy component not dominant in the past. At high z_s, we have an asymptotic value 95% confidence level (c.l.) of $z_b > 3.0$. Also plotted on the figure are the likelihood contours derived by a combined CMB+SN-Ia analysis. The inclusion of the SN-Ia data improves the constraints to $z_b > 5$ and $z_s > 100$ at $1-\sigma$. SN-Ia data are insensitive to variations in z_s since they are probing only redshifts $z < 1.5$, but provide anyway complementary constraints on z_b and the matter density Ω_m.

This is better explained in Fig. 4, where we overimpose the likelihood contours in the (Ω_m, z_b) plane from CMB and SN-Ia analysis. As we can see, the current SN-Ia data does not provide evidence for dark energy evolution. However, low values of z_b are compatible with the SN-Ia data if one decreases the amount of the matter component. The SN-Ia data is consistent with $\Omega_m = 0$ but in this case the dark energy model behaves like a unified dark energy model with transition redshift $z_b \sim 0.5$. On the other hand, as we can see from the plot, lower values of z_b are compatible with CMB data if one increases the matter density. This is easily explained from the fact that a lower z_b results in higher values for the effective dark energy equation of state. The direction of degeneracy in the plot in the case of the CMB data is therefore only a consequence of the geometrical degeneracy present in angular diameter distance data at high redshift. The lower limit on z_b from the CMB data comes mainly from our assumptions on the possible values of the Hubble parameter. Combining the CMB and SN-Ia further breaks this degeneracy, improving the lower limit on z_b and excluding at high significance a unified dark energy model with $\Omega_m = 0$.

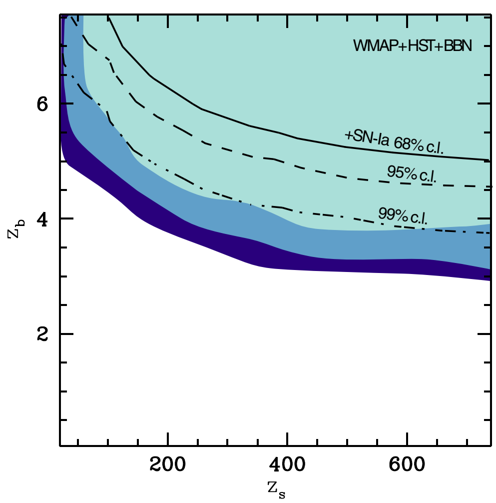

FIG. 3 (color online). Likelihood probability contours at 68% (light gray), 95% (medium gray) and 99% (dark gray) in the z_s-z_b plane from WMAP. The solid, dashed, and dot-dashed lines are the 68%, 95%, and 99% from a WMAP + SN-Ia analysis.

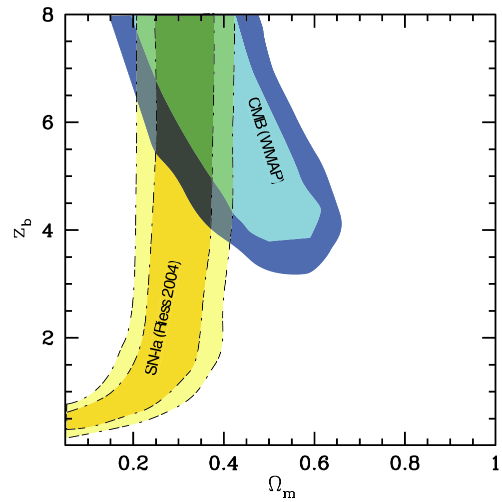

FIG. 4 (color online). The 68% and 95% likelihood probability contours in the Ω_m-z_b plane from WMAP and SN-Ia.

V. CONCLUSIONS

A phenomenological *scaling dark energy* model is discussed in this paper matching it with the current CMB and SN-Ia data to identify the signatures of a possible cosmological evolution of dark energy, as expected and predicted in several theories. We do not take into account particular scalar field or quintessence models but discuss how the state equation of cosmic fluid, depending on redshift, scales with respect to the epoch passing from a radiation regime to a dark-energy-matter-dominated era. The approach is extremely general since the dynamical behavior which we discuss should be the one expected for most of the models present in literature.

We found that the current data does not show evidence for cosmological evolution of dark energy, providing the

68% c.l. bounds $z_b > 5.0$ and $z_s > 100$ constraining the presence of scaling dark energy in the Universe. A simple but theoretically flawed cosmological constant still provides a good fit to the data (see also the discussion in [9]).

Interesting correlations between the parameters used in the analysis of single data sets are present. For example, lower values of the matter density ($\Omega_m \sim 0.1$) could hint for an evolution in the SN-Ia data, but these values are ruled out by CMB measurements. Models which have a non-negligible energy contribution at the recombination and behave as a nonrelativistic component (dust) are ruled out from our combined analysis.

Dark energy models with a subdominant contribution to the overall energy density of the Universe for most of the cosmological evolution are clearly preferred. While this condition is easy to achieve for many models based on scalar fields or topological defects (see, e.g., [27]), the quantity of information we can hope to extract from future data about dark energy is more limited.

Finally, we would like to underline more that the validity of our results is limited to the class of scaling dark energy models with a slowly varying behavior of the equation of state at late time. Our parametrization does not account for rapid variations which can be present in some scaling models [28]. Moreover not all quintessence models show scaling behaviors. For instance, the Inverse Power-Law potential and the SUGRA inspired potentials only have tracking solutions.

However, our approach could be further improved and extended to more general scaling solutions. In this respect, the incoming wide cosmological surveys such as PLANCK or SNAP will provide essential data.

ACKNOWLEDGMENTS

We wish to thank Luca Amendola, Vincenzo Cardone, and Antonio Troisi for useful discussions.

[1] C. Wetterich, Nucl. Phys. **B302**, 668 (1988).
[2] R. R. Caldwell, R. Dave, and P. J. Steinhardt, Phys. Rev. Lett. **80**, 1582 (1998).
[3] C. Armendariz-Picon, T. Damour, and V. Mukhanov, Phys. Lett. B **458**, 209 (1999).
[4] T. Chiba, T. Okabe, and M. Yamaguchi, Phys. Rev. D **62**, 023511 (2000).
[5] L. Amendola, Phys. Rev. D **62**, 043511 (2000).
[6] S. Capozziello, Int. J. Mod. Phys. D **11**, 483 (2002); S. Capozziello, V. F. Cardone, S. Carloni, and A. Troisi, Int. J. Mod. Phys. D **12**, 1969 (2003); S. M. Carroll, V. Duvvuri, M. Trodden, and M. S. Turner, Phys. Rev. D **70**, 043528 (2004); S. Nojiri and S. D. Odintsov, Phys. Rev. D **68**, 123512 (2003); E. E. Flanagan, Phys. Rev. Lett. **92**, 071101 (2004); G. Allemandi, A. Borowiec, and M. Francaviglia, Phys. Rev. D **70**, 043524 (2004).
[7] I. Zlatev, L. Wang, and P. J. Steinhardt, Phys. Rev. Lett. **82**, 896 (1999); Phys. Rev. D **59**, 123504 (1999).
[8] J. J. Halliwell, Phys. Lett. B **185**, 341 (1987); J. Barrow, Phys. Lett. B **187**, 12 (1987); B. Ratra and P. Peebles, Phys. Rev. D **37**, 3406 (1988); C. Wetterich, Astron. Astrophys. **301**, 321 (1995).
[9] C. Rubano, P. Scudellaro, E. Piedipalumbo, S. Capozziello, and M. Capone, Phys. Rev. D **69**, 103510 (2004).
[10] C. L. Bennett *et al.*, Astrophys. J. Suppl. Ser. **148**, 1 (2003).
[11] Supernova Search Team Collaboration, A. G. Riess *et al.*, Astrophys. J. **607**, 665 (2004).
[12] U. Alam, V. Sahni, and A. A. Starobinsky, J. Cosmol. Astropart. Phys. 06 (2004) 008.
[13] D. Huterer and A. Cooray, astro-ph/0404062.
[14] V. F. Cardone, A. Troisi, and S. Capozziello, Phys. Rev. D **69**, 083517 (2004).
[15] P. S. Corasaniti, M. Kunz, D. Parkinson, E. J. Copeland, and B. A. Bassett, astro-ph/0406608 [Phys. Rev. D (to be published)]; R. R. Caldwell and M. Doran, Phys. Rev. D **69**, 103517 (2004).
[16] A. Melchiorri, L. Mersini, C. J. Odman, and M. Trodden, Phys. Rev. D **68**, 043509 (2003).
[17] T. Chiba, N. Sugiyama, and T. Nakamura, Mon. Not. R. Astron. Soc. **289**, L1 (1997); T. Chiba, N. Sugiyama, and T. Nakamura, Mon. Not. R. Astron. Soc. **301**, 72 (1998).
[18] C. P. Ma and E. Bertschinger, Astrophys. J. **455**, 7 (1995).
[19] Wands *et al.*, Phys. Rev. D **62**, 043527 (2000).
[20] Bartolo, Corasaniti, Liddle, and Malquarti, Phys. Rev. D **70**, 043532 (2004).
[21] U. Seljak and M. Zaldarriaga, Astrophys. J. **469**, 437 (1996).
[22] R. Bean and A. Melchiorri, Phys. Rev. D **65**, 041302 (2002).
[23] R. Bowen, *et al.*, Mon. Not. R. Astron. Soc. **334**, 760 (2002).
[24] W. Freedman *et al.*, Astrophys. J. **553**, 47 (2001).
[25] M. Kunz, P. S. Corasaniti, D. Parkinson, and E. J. Copeland, Phys. Rev. D **70**, 041301 (2004).
[26] L. Verde *et al.*, Astrophys. J. Suppl. Ser. **148**, 195 (2003).
[27] L. Conversi, A. Melchiorri, L. Mersini, and J. Silk, Astropart. Phys. **21**, 443 (2004).
[28] Barreiro, Nunes, and Copeland, Phys. Rev. D **61**, 127301 (2000); Albrecht and Skordis, Phys. Rev. Lett. **84**, 2076 (2000).